Acceleration of particles by black hole with gravitomagnetic charge immersed in magnetic field

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Abstract

The collision of test charged particles in the vicinity of an event horizon of a weakly magnetized non-rotating black hole with gravitomagnetic charge has been studied. The presence of the external magnetic field decreases the innermost stable circular orbits (ISCO) radii of charged particles. The opposite mechanism occurs when there is nonvanishing gravitomagnetic charge. For a collision of charged particle moving at ISCO and the neutral particle falling from infinity the maximal collision energy can be decreased by gravitomagnetic charge in the presence of external asymptotically uniform magnetic field.

Keywords Particle motion Acceleration mechanism NUT spacetime

1 Introduction

At present there is no any observational evidence for the existence of gravitomagnetic monopole, i.e. so-called NUT (Newman et al. 1963) parameter or *magnetic mass*. Therefore study of the motion of the test particles and particle acceleration mechanisms in NUT spacetime may provide new tool for studying new important general relativistic effects which are associated with nondiagonal components of the metric tensor and have no Newtonian analogues (See, e.g. Nouri-Zonoz and Lynden-Bell (1997); Lynden-Bell and Nouri-Zonoz (1998); Nouri-Zonoz et. al (1999); Nouri-Zonoz (2004); Kagramanova et al. (2008, 2010); Morozova and Ahmedov (2009) where solutions for electromagnetic waves and interferometry in spacetime with NUT parameter have been studied.). Kerr-Taub-NUT spacetime with Maxwell and dilation fields is recently investigated by authors of the paper Aliev et al. (2008). In our preceding papers Morozova et al. (2008); Abdujabbarov et al. (2008) we have studied the plasma magnetosphere around a rotating, magnetized neutron star and charged particle motion around compact objects immersed in external magnetic field in the presence of the NUT parameter. The Penrose process in the spacetime of rotating black hole with nonvanishing gravitomagnetic charge has been considered in Abdujabbarov et al. (2011). The electromagnetic field of the relativistic star with nonvanishing gravitomagnetic charge has been considered by Ahmedov et. al (2012).

Astrophysical processes which may produce high energy radiation near a rotating black hole horizon attract more attention in recent publications. Such processes connected with the Penrose effect (Penrose 1969) have been studied long time ago by Piran et. al (1975); Piran and Shaham (1977a,b). Recently Banados et al. (2009) (BSW) demonstrated that for an extremely rotating black hole such collisions can produce particles of high center-of-mass energy. The results of Banados et al. (2009) have been commented by Berti et al. (2009) where authors concluded that astrophysical limitations on the maximal spin, backreaction effects and sensitivity to the initial conditions impose severe limits on the likelihood of such accelerations. The above mentioned discussions forces

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us to study the particle acceleration in the spacetime of a black hole with nonvanishing gravitomagnetic charge. The acceleration of particles, circular geodesics, accretion disk, and high-energy collisions in the Janis-Newman-Winicour spacetime have been considered by Patil and Joshi (2012); Chowdhury et al. (2012).

The aim of this paper is to show that a similar effect of particle collision with high center-of-mass energy is also possible when a black hole is non-rotating (or slowly rotating) provided there exists magnetic field in its exterior and in the presence of the gravitomagnetic charge. It is very interesting to study the electromagnetic fields and particle motion in NUT space with the aim to get new tool for studying new important general relativistic effects. This observation might be interesting since there exist both theoretical and experimental indications that such a magnetic field must be present in the vicinity of black holes. In what follows we assume that this field is weak and its energy-momentum does not modify the background black hole geometry. For a black hole of mass M this condition holds if the strength of magnetic field satisfies the condition (Piotrovich et.al 2011; Frolov 2012)

$$B \ll B_{max} = \frac{c^4}{G^{3/2}M_\odot}\left(\frac{M_\odot}{M}\right) \sim 10^{19}\frac{M_\odot}{M}\text{Gauss}\,. \quad (1)$$

We call such black holes *weakly magnetized*. One can expect that the condition (1) is satisfied both for stellar mass and supermassive black holes.

The paper is organized as follows. Section II is devoted to study of electromagnetic field and charged particle motion in the magnetized black holes with NUT parameter, with the main focus on the properties of their ISCOs. Particles collisions in the vicinity of a weakly magnetized black hole with nonvanishing gravitomagnetic charge have been studied in Section III. The concluding remarks and discussions are presented in Section IV.

Throughout the paper, we use a space-like signature $(-,+,+,+)$ and a system of units in which $G = 1 = c$ (However, for those expressions with an astrophysical application we have written the speed of light explicitly.). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

2 Charged particle motion around black hole with nonvanishing NUT charge

Here we will consider a charged particle motion in the vicinity of a black hole of mass M in the presence of an external static axisymmetric and uniform at the spatial infinity magnetic field. The appropriate spacetime metric has the following form (see Dadhich and Turakulov 2002; Bini et al. 2003):

$$\begin{aligned} ds^2 &= -\frac{\Delta}{\Sigma}\,dt^2 + 4\frac{\Delta}{\Sigma}\,l\cos\theta dt d\varphi + \frac{1}{\Sigma}\,(\Sigma^2\sin^2\theta - \\ &\quad - 4\Delta l^2\cos^2\theta)d\varphi^2 + \frac{\Sigma}{\Delta}\,dr^2 + \Sigma d\theta^2\ , \end{aligned} \quad (2)$$

where parameters Σ and Δ are defined as

$$\Sigma = r^2 + l^2, \qquad \Delta = r^2 - 2Mr - l^2\ ,$$

where l is the gravitomagnetic monopole momentum. Here we will exploit the existence in this spacetime of a timelike Killing vector $\xi^\alpha_{(t)} = \partial x^\alpha/\partial t$ and spacelike one $\xi^\alpha_{(\varphi)} = \partial x^\alpha/\partial\varphi$ being responsible for stationarity and axial symmetry of geometry, such that they satisfy the Killing equations

$$\xi_{\alpha;\beta} + \xi_{\beta;\alpha} = 0 \quad (3)$$

which gives a right to write the solution of vacuum Maxwell equations $\Box A^\mu = 0$ for the vector potential A_μ of the electromagnetic field in the Lorentz gauge in the simple form

$$A^\alpha = C_1\xi^\alpha_{(t)} + C_2\xi^\alpha_{(\varphi)}. \quad (4)$$

The constant $C_2 = B/2$, where gravitational source is immersed in the uniform magnetic field $\mathbf{B}$ being parallel to its axis of rotation. The value of the remaining constant C_1 can be easily calculated from the asymptotic properties of spacetime (2) at the infinity. Indeed in order to find the remaining constant one can use the electrical neutrality of the black hole

$$\begin{aligned} 4\pi Q &= \frac{1}{2}\oint F^{\alpha\beta}{}_{*}dS_{\alpha\beta} = C_1\oint \Gamma^\alpha_{\beta\gamma}\tau_\alpha m^\beta\xi^\gamma_{(t)}(\tau k)dS \\ &\quad + \frac{B}{2}\oint \Gamma^\alpha_{\beta\gamma}\tau_\alpha m^\beta\xi^\gamma_{(\phi)}(\tau k)dS = 0\ , \end{aligned} \quad (5)$$

evaluating the value of the integral through the spherical surface at the asymptotic infinity. Here the equality $\xi_{\beta;\alpha} = -\xi_{\alpha;\beta} = -\Gamma^\gamma_{\alpha\beta}\xi_\gamma$ following from the Killing equation was used, and element of an arbitrary 2-surface $dS^{\alpha\beta}$ is represented in the form (see, e.g. Ahmedov and Rakhmatov 2003)

$$dS^{\alpha\beta} = -\tau^\alpha \wedge m^\beta(\tau k)dS + \eta^{\alpha\beta\mu\nu}\tau_\mu n_\nu\sqrt{1+(\tau k)^2}dS\ , \quad (6)$$

and the following couples

$$m_\alpha = \frac{\eta_{\lambda\alpha\mu\nu}\tau^\lambda n^\mu k^\nu}{\sqrt{1+(\tau k)^2}},$$
$$n_\alpha = \frac{\eta_{\lambda\alpha\mu\nu}\tau^\lambda k^\mu m^\nu}{\sqrt{1+(\tau k)^2}},$$
$$k^\alpha = -(\tau k)\tau^\alpha + \sqrt{1+(\tau k)^2}\eta^{\mu\alpha\rho\nu}\tau_\mu m_\rho n_\nu$$

are established between the triple $\{\mathbf{k}, \mathbf{m}, \mathbf{n}\}$ of vectors, n^α is normal to 2-surface, space-like vector m^α belongs to the given 2-surface and is orthogonal to the four-velocity of observer τ^α, a unit spacelike four-vector k^α belongs to the surface and is orthogonal to m^α, dS is invariant element of surface, $\wedge$ denotes the wedge product, $_*$ is for the dual element, $\eta_{\alpha\beta\gamma\delta}$ is the pseudo-tensorial expression for the Levi-Civita symbol $\epsilon_{\alpha\beta\gamma\delta}$.

Then one can insert $\tau_0 = -(1-M/r)$, $m^1 = (1-M/r)$, and asymptotic values for the Christoffel symbols $\Gamma^0_{10} = M/r^2$ and $\Gamma^0_{13} = -l(1-2M/r)\cos\theta/r$ in the flux expression (5) and get the value of constant $C_1 = 0$. Parameter l does not affect on constant C_1 because the integral $\int_0^\pi \cos\theta\sin\theta d\theta = 0$ vanishes.

Thus the 4-vector potential A_μ of the electromagnetic field will take the following form

$$A_0 = -\frac{\Delta}{\Sigma}Bl\cos\theta \tag{7}$$
$$A_3 = \frac{1}{2}\Sigma^2\sin^2\theta - 2\Delta l^2\cos^2\theta \tag{8}$$

The orthonormal components of the electromagnetic fields measured by zero angular momentum observers (ZAMO) with four velocity components

$$(\tau^\alpha)_{\rm ZAMO} \equiv \left(\sqrt{\frac{\mathcal{R}}{\Delta\Sigma\sin^2\theta}}, 0, 0, \frac{2\Delta l\cos\theta}{\sqrt{\Delta\Sigma\mathcal{R}\sin^2\theta}}\right) \tag{9}$$
$$(\tau_\alpha)_{\rm ZAMO} \equiv \left(\sqrt{\frac{\Delta\Sigma\sin^2\theta}{\mathcal{R}}}, 0, 0, 0\right) \tag{10}$$

are given by expressions

$$E^{\hat r} = -\frac{Brl}{\sqrt{\mathcal{R}}}\left(1-\frac{M}{r}\right)\sin 2\theta\ , \tag{11}$$
$$E^{\hat\theta} = \frac{Bl}{\Sigma^2}\sqrt{\frac{\Delta}{\mathcal{R}}} \times\left[\Sigma^2 + \left(\Sigma^2 - 2\Delta l^2\cos\theta\right)\frac{2\cos\theta}{\sin^2\theta}\right]\sin^2\theta\ , \tag{12}$$
$$B^{\hat r} = \frac{B\tan\theta}{\Sigma\sqrt{\mathcal{R}}}\left(\mathcal{R}-\Sigma^2\right)\ , \tag{13}$$
$$B^{\hat\theta} = \frac{Br}{\Sigma^2}\sqrt{\frac{\Delta}{\mathcal{R}}}\cos^2\theta \times\left\{\left[\Delta - \Sigma\left(1-\frac{M}{r}\right)\right]4l^2 + \Sigma^2\tan^2\theta\right\}\ , \tag{14}$$

where the following notation has been used:

$$\mathcal{R} = \Sigma^2\sin^2\theta - 4\Delta l^2\cos^2\theta.$$

Astrophysically it is interesting to know the limiting cases of expressions (11) - (14), for example in either linear or quadratic approximation $\mathcal{O}(a^2/r^2, l^2/r^2)$ in order to give physical interpretation of possible physical processes near the slowly rotating relativistic compact objects, where they take the following form:

$$E^{\hat r} = \frac{2Bl\cos\theta}{r}\left(1-\frac{3M}{r}\right)\ , \tag{15}$$
$$E^{\hat\theta} = \frac{Bl\sin\theta}{r}\left(1+\frac{2\cos\theta}{\sin^2\theta}\right)\ , \tag{16}$$
$$B^{\hat r} = B\cos\theta\left[1+2\frac{l^2}{r^2}\frac{1}{\sin^2\theta}\right]\ , \tag{17}$$
$$B^{\hat\theta} = B\sin\theta\Big[1-\frac{M}{r}+\frac{1}{16r^2\sin^2\theta} \times\left(4l^2-4M^2+4(7l^2+M^2)\cos 2\theta\right)\Big]\ . \tag{18}$$

In the limit of flat spacetime, i.e. for $M/r\to 0$ and $l^2/r^2 \to 0$, expressions (15)-(18) give

$$E^{\hat r} = E^{\hat\theta} \to 0\ , \tag{19}$$

$$B^{\hat r} \to B\cos\theta, \qquad B^{\hat\theta} \to B\sin\theta\ . \tag{20}$$

As expected, expressions (19)-(20) coincide with the solutions for the homogeneous magnetic field in Newtonian spacetime.

The dynamical equation for a charged particle motion can be written as

$$m\frac{du^\mu}{d\tau} = qF^\mu_{\ \nu}u^\nu\ , \tag{21}$$

where τ is the proper time, u^μ is the particle 4-velocity, $u^\mu u_\mu = -1$, q and m are its charge and mass, respectively. $F_{\alpha\beta} = A_\beta,\alpha - A_\alpha,\beta$ is the antisymmetric tensor of the electromagnetic field, which has the following four independent components

$$F_{01} = \frac{B}{\Sigma^2}2l[(r-M)-\Delta r]\cos\theta\ ,$$
$$F_{02} = \frac{B}{\Sigma}\Delta l\sin\theta\ ,$$
$$F_{13} = -\frac{B}{\Sigma^2}8l^2[(r-M)\Sigma-\Delta r]\cos^2\theta + Br\sin^2\theta\ ,$$
$$F_{23} = \frac{B}{\Sigma}(4l^2\Delta + \frac{1}{2}\Sigma^2)\sin 2\theta\ .$$

While considering the charged particle motion around black hole immersed in the magnetic field it is easy

to use two conserved quantities associated with the Killing vectors: the energy $\mathcal{E} > 0$ and the generalized azimuthal angular momentum $\mathcal{L} \in (-\infty, +\infty)$:

$$\begin{aligned}
\mathcal{E} &\equiv -\xi^{\mu}_{(t)} P_\mu = \frac{m\Delta}{\Sigma}\left(\frac{dt}{d\tau} + 4l\cos\theta\frac{d\varphi}{d\tau} + \frac{q}{m}Bl\cos\theta\right), && (22)\\
\mathcal{L} &\equiv \xi^{\mu}_{(\phi)} P_\mu = -4ml\frac{\Delta}{\Sigma}\cos\theta\frac{dt}{d\tau} + (\Sigma\sin^2\theta - \\
&\qquad - 4l^2\frac{\Delta}{\Sigma}\cos^2\theta)\left(m\frac{d\varphi}{d\tau} + \frac{qB}{2}\right). && (23)
\end{aligned}$$

Here $P_\mu = m\,u_\mu + qA_\mu$ is the generalized 4-momentum of the particle. It was first shown by Zimmerman and Shahid (1989) for spherical symmetric case (NUT spacetime) and later by Bini et al. (2003) for axial symmetric case (Kerr-Taub-NUT spacetime) that the orbits of the test particles are confined to a cone with the opening angle θ given by $\cos\theta = 2\mathcal{E}l/\mathcal{L}$. It also follows that in this case the equations of motion on the cone depend on l only via l^2 (Bini et al. 2003; Abdujabbarov et al. 2008). The main point is that the small value for the upper limit for gravitomagnetic moment has been obtained by comparing theoretical results with experimental data as (i) $l \leq 10^{-24}$ from the gravitational microlensing (Rahvar and Habibi 2004), (ii) $l \leq 1.5\cdot 10^{-18}$ from the interferometry experiments on ultracold atoms (Morozova and Ahmedov 2009), (iii) and similar limit has been obtained from the experiments on Mach-Zehnder interferometer (Kagramanova et al. 2008). Due to the smallness of the gravitomagnetic charge let us consider the motion in the quasi-equatorial plane when the motion in θ direction changes as $\theta = \pi/2 + \delta\theta(t)$, where $\delta\theta(t)$ is the term of first order in l, then it is easy to expand the trigonometric functions as $\sin\theta = 1 - \delta\theta^2(t)/2 + \mathcal{O}(\delta\theta^4(t))$ and $\cos\theta = \delta\theta(t) - \mathcal{O}(\delta\theta^3(t))$. Neglecting the small terms $\mathcal{O}(\delta\theta^2(t))$, one can easily obtain the geodesic equation in the following form

$$\begin{aligned}
\frac{dt}{d\tau} &= \frac{\mathcal{E}}{m}\frac{\Sigma}{\Delta}\,, && (24)\\
\frac{d\varphi}{d\tau} &= \frac{\mathcal{L}}{m\Sigma} - \frac{qB}{2m}\,, && (25)\\
\frac{dr}{d\tau} &= \left(\frac{\mathcal{E}}{m} - U\right)\,, && (26)
\end{aligned}$$

where the effective potential is

$$U = \frac{\Delta}{\Sigma}\left(1 + \Sigma\chi^2\right)\,, \qquad (27)$$

and

$$\chi = \frac{\mathcal{L}}{m\Sigma} - \frac{qB}{2m}\,. \qquad (28)$$

In the expressions (24)–(26) the terms being proportional to the second and higher orders of the small parameter $\delta\theta$ are neglected. In the Fig. 1 the radial dependence of the effective potential of the radial motion of the charged particle is presented for the different values of dimensionless gravitomagnetic parameter l/M and magnetic parameter $b = qBM/m$.

From the Fig. 1 one can obtain the behavior of the charged particle motion in the presence of both the gravitomagnetic charge and magnetic field. In the presence of the gravitomagnetic charge the minimum of the effective potential shifts towards the observer at the infinity which means that the orbits of the charged parti[redacted]es may become unstable. The minimum value of the radius of the stable circular orbits increases. The influence of the magnetic field has the opposite effect: the presence of the magnetic field decreases the minimum value of the circular orbits radius and charged particles may come much closer to the central object.

In order to study innermost stable circular orbits (ISCO) we use its first and second derivatives of the U and equalize them to zero:

$$\begin{aligned}
U' &= -\frac{2\Delta r(1 + 2\mathcal{L}\chi)}{\Sigma^2} \\
&\quad + \frac{1}{r}\left(1 + \frac{\Delta}{\Sigma}\right)\left(1 + \Sigma\chi^2\right) = 0 && (29)\\
U'' &= \frac{8\Delta L^2 r^2}{\Sigma^4} + \frac{2}{\Sigma}\left(\frac{4\Delta r^2}{\Sigma^2} - \frac{3\Delta}{\Sigma} - 2\right) \\
&\quad \times(1 + 2\mathcal{L}\chi) + + \frac{2}{\Sigma}\left(1 + \Sigma\chi^2\right) = 0 && (30)
\end{aligned}$$

Now we have two equations with three unknown quantities $\mathcal{L}$, r and χ. Solving the equation (29) we derive $\mathcal{L}$ in terms r and χ

$$\mathcal{L} = \frac{-2\Delta r^2 + \Delta\Sigma + \Sigma^2 + \Delta\Sigma^2\chi^2 + \Sigma^3\chi^2}{4\Delta r^2\chi}\,. \qquad (31)$$

Substituting this equation into (30) one can easily obtain the equation expressing χ in terms of r

$$\chi_\pm = \pm\left[\frac{K}{\Sigma A}\left(1 \pm \sqrt{1 - \frac{A\Sigma}{K^2}\left(\Delta + \Sigma - \frac{2r^2}{\Sigma}\Delta\right)^2}\right)\right]^{\frac{1}{2}}, \qquad (32)$$

where

$$\begin{aligned}
A &= 8\Delta^2 r^2 - 5\Delta^2\Sigma + 12\Delta r^2\Sigma - 8\Delta\Sigma^2 - 3\Sigma^3,\\
K &= -2\Delta^2 r^2 + 2\Delta^2\Sigma - 4\Delta r^2\Sigma + 3\Delta\Sigma^2 + \Sigma^3.
\end{aligned}$$

The signs $\pm$ correspond to co-rotating and counter-rotating particle orbits, respectively. The dependence of the $\chi_\pm$ from the ISCO radius are shown in Fig. 2. The shift of the shape of χ_+ to the right direction

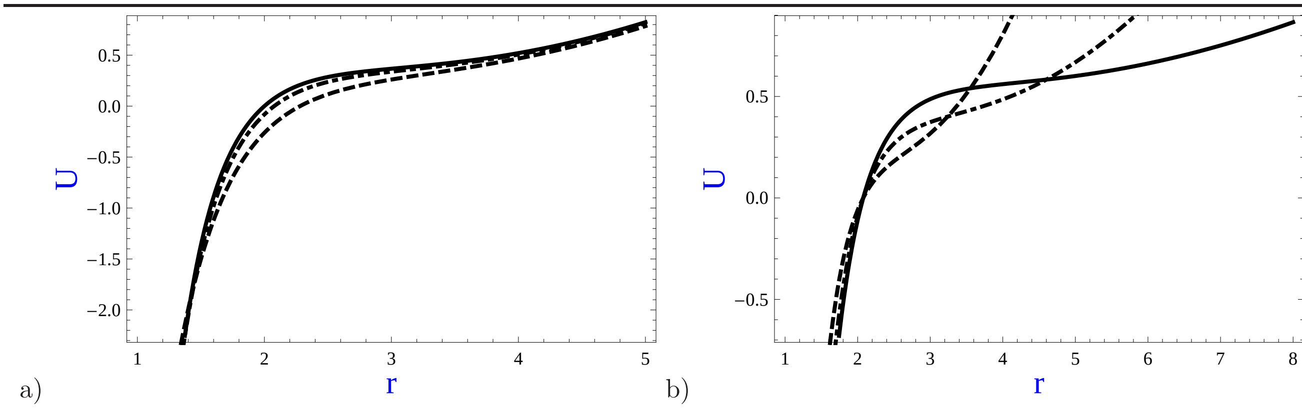

Fig. 1 The radial dependence of the effective potential of radial motion of the charged particle for the different values of the gravitomagnetic charge (a): $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line) and for the different values of the dimensionless magnetic parameter $b = qBM/m$ (b): $b = 0.2$ (solid line), $b = 0.4$ (dot-dashed line), $b = 0.8$ (dashed line).

with increasing the gravitomagnetic charge corresponds to increasing of ISCO radius in the presence of NUT charge.

From the Fig. 2 one can see that χ_- function steadily increase when particle approach to the black hole horizon and go to infinity in the case when gravitomagnetic charge vanishes. In the case of nonvanishing NUT-parameter, the function χ_- falls in the point of singularity. But by reason of that jumps are located inside the horizon, the effects that take place there are not observable relatively to detached observer and cannot be interpreted as some full physical theory. In other hand the spacetime of nonrotating black hole allows us to find analytic continuations of our theories inside the black hole horizon up to a point of singularity by excepting it.

Consider a circular motion of the charged particle in the presence of NUT charge. The effective potential has minimum value at the radius of such orbit. The 4-momentum of a test particle at the circular orbit of radius r is

$$p^\mu = m\gamma(e^\mu_{(t)} + v e^\mu_{(\phi)}) \,, \tag{33}$$

$$e^\mu_{(t)} = (\Sigma/\Delta)^{1/2}\xi^\mu_{(t)} = (\Sigma/\Delta)^{1/2}\delta^\mu_t \,, \tag{34}$$

$$e^\mu_{(\phi)} = \Sigma^{-1/2}\xi^\mu_{(\phi)} = \Sigma^{-1/2}\delta^\mu_\phi \,. \tag{35}$$

Here v (which can be both positive and negative) is a velocity of the particle with respect to a rest frame, and γ is the Lorentz gamma factor. From normalization condition $\mathbf{p}^2 = -m^2$ one has $\gamma = (1 - v^2)^{-1/2}$. For $q > 0$ the Lorentz force acting on a particle with $v > 0$ is *repulsive* (i.e. directed outwards the black hole), while for $v < 0$ it is *attractive*.

Using relation $d\phi/d\tau = v\gamma/r$ and (25) with $\theta = \pi/2$ one gets

$$\frac{v\gamma}{r} = \chi \,. \tag{36}$$

This relation allows us to find

$$\gamma = \sqrt{1 + r^2\chi^2} \quad \text{and} \quad v = \frac{r\chi}{\sqrt{1 + r^2\chi^2}} \,. \tag{37}$$

Using (37) one can find the values of the velocity $v_\pm$ and γ-factor $\gamma_\pm$. Fig. 3 shows the velocity of a particle at the ISCO as a function of its radius, while Fig. 4 shows the dependence of $\gamma_\pm$ from $r_{\rm ISCO}$.

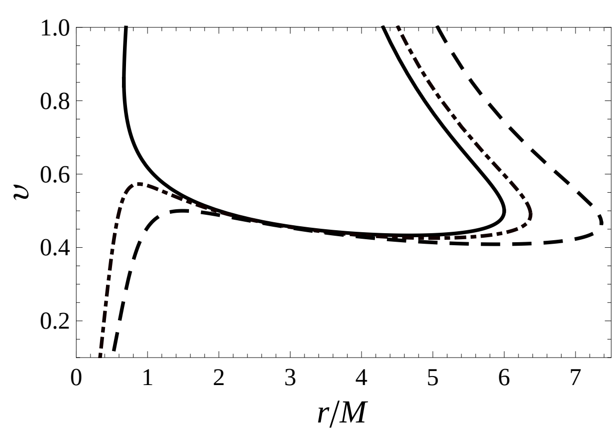

Fig. 3 Velocity of the particles at $r_{\rm ISCO}$ as a function of its radius for different values of the gravitomagnetic charge: $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line).

The Fig. 3 shows the dependence of the velocity of the particle at ISCO. Since there are two values of velocity for the same radius one can interpret them as

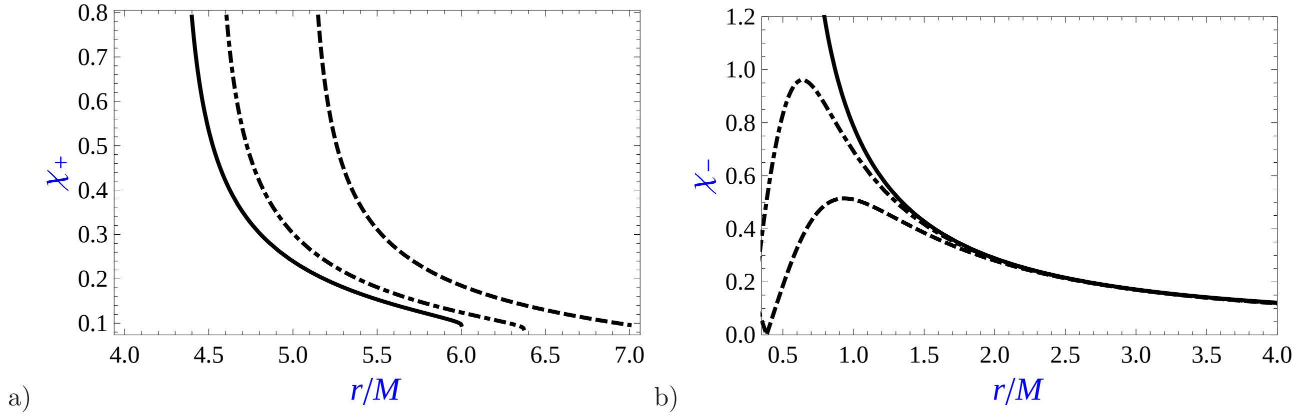

Fig. 2 χ_+ (a) and χ_- (b) as a function of the ISCO radius for different values of the gravitomagnetic charge: $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line).

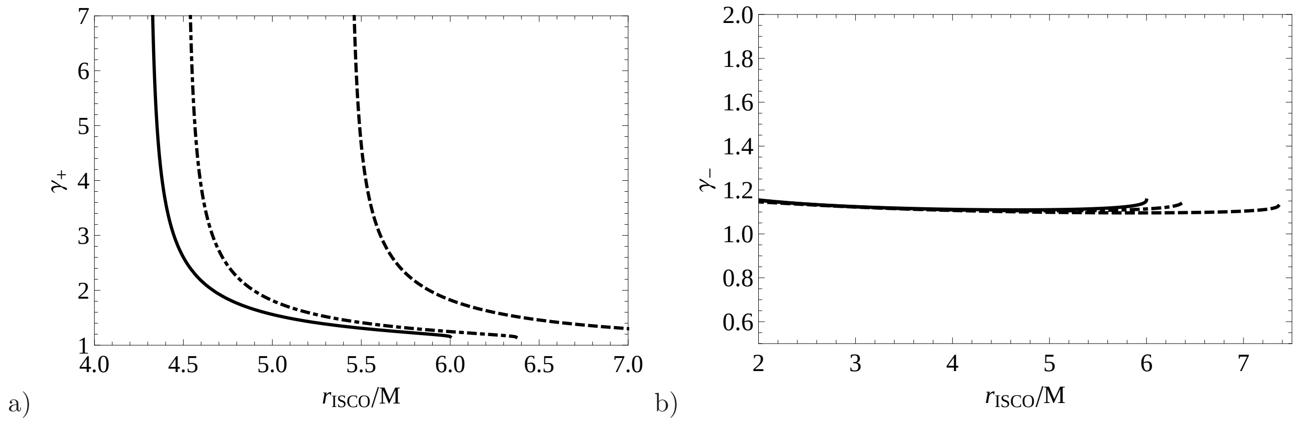

Fig. 4 γ_+ (a) and γ_- (b) as a function of the ISCO radius for different values of the gravitomagnetic charge: $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line).

two values of velocity which correspond for two opposite directions of motion of the particles. Since charged particle and magnetic field interaction depends on the velocity direction there should be two values $v_\pm$ for each $r_{\rm ISCO}$. Furthermore the absence of external magnetic field (right border of the plots) one can obtain only one solution for the velocity at ISCO. One should mention that in the non-relativistic case one can get the Keplerian velocity profile.

In the next step we will consider the case, when a neutral particle freely falling from infinity collides with a charged particle revolving at the circular orbit near a weakly magnetized black hole with nonvanishing gravitomagnetic charge. One can denote by $\mathbf{p}$ the momentum of this particle, and by m and q its mass and charge. Denote by μ the mass of a freely falling particle, and by $\mathbf{k}$ its 4-momentum. At the moment of collision the 4-momentum is

$$\mathbf{P} = \mathbf{p} + \mathbf{k} \,, \tag{38}$$

and the corresponding center-of-mass energy $E_{\rm c.m.}$ is

$$E_{\rm c.m.} = m^2 + \mu^2 - 2(\mathbf{p}, \mathbf{k}) \,. \tag{39}$$

Using the equation of particle motion around black hole with nonvanishing gravitomagnetic charge (24)–(26) one can obtain the following relation for the center-of-mass energy of two colliding particles for the weak magnetic field approximation:

$$\frac{E_{\rm c.m.}}{m} \simeq 0.3\sqrt{\frac{96 - l^2}{\sqrt{8 + l^2}}} b^{1/4} \,. \tag{40}$$

In the Table 1 the dependence of the ISCO radius and the center of mass energy of colliding charged particles have peen shown. From the results on can conclude that gravitomagnetic charge correction prevents the particle from the infinite acceleration.

From the obtained result one can observe that the presence of the gravitomagnetic charge will decrease the value of the center of mass energy. The role of the magnetic field in particle accelerating process is to decrease

Table 1 The dependence of the center of mass energy and ISCO radii from the magnetic parameter b for the different values of the specific gravitomagnetic charge l/M:

l/M	0	0.2	0.4	0.6	0.8	1.0	4.52
$r_{\rm ISCO}$	$2+0.58b^{-1}$	$2.02+0.58b^{-1}$	$2.08+0.59b^{-1}$	$2.17+0.60b^{-1}$	$2.28+0.61b^{-1}$	$2.41+0.62b^{-1}$	$5.63+0.74b^{-1}$
$E_{\rm c.m.}/m$	$1.747b^{1/4}$	$1.745b^{1/4}$	$1.738b^{1/4}$	$1.725b^{1/4}$	$1.708b^{1/4}$	$1.688b^{1/4}$	$1.129b^{1/4}$

the innermost stable circular orbits radii. As particles come closer to the black hole horizon their energy at infinity is going to increase. The role of the gravitomagnetic charge in this process is opposite: the presence of the gravitomagnetic charge increase the radii of ISCO.

3 Conclusion

In this report we have obtained the expressions for the energy and angular momentum as well as ISCO of the charged particle in the vicinity of the black hole in presence of gravitomagnetic charge and exterior magnetic field.

Recently Banados et al. (2009) underlined that a rotating black hole can, in principle, accelerate the particles falling to the central black hole to arbitrary high energies. Frolov (2012) has shown that the magnetic field could play a role of charged particle accelerator near the nonrotating black hole. Because of some mechanisms such as astrophysical limitations on the maximum spin, backreaction effects, upper limit for magnetic field, and sensitivity to the initial conditions, there appears to be some upper limit for the center of mass energy of the infalling particles. One of the mechanisms offered in this paper is appearing due to the gravitomagnetic charge correction which prevents the particle from the infinite acceleration.

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